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Image Signal-to-Noise Ratio in Decibels – Part 1

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Turn up the signal
Wipe out the noise
 - *Signal to Noise*, Peter Gabriel

The Decibel (dB) is a logarithmic way to describe a ratio with respect to a reference level. That part is easy. Now when it comes to expressing Signal-to-Noise Ratio (SNR), and particularly imaging SNR in dB, a couple of areas of confusion arise.

The original decibel (dB) is the logarithmic unit used to measure sound level. It is useful because several human perceptions follow an approximately power-law or logarithmic relationship with a corresponding physical measure (e.g. for loudness, lightness).

In electrical engineering, the SNR is often defined in terms of the ratio of power, *P*, in watts

$$SNR = \frac{P_{signal}}{P_{noise}}. \quad (1)$$

Note, however, that in many cases an information-bearing signal would be expressed as a voltage (volts, v) rather than watts (W). Further, by Ohm's Law

$$1\text{W} = \text{v}^2 / \Omega , \quad (2)$$

where Ω is the circuit resistance over which the voltage is applied or measured. So we see that a signal in watts is proportional to *square* of the corresponding signal in volts, given that for simplicity we assume that resistance does varying with 'signal'. The voltage is the signal *amplitude*.

We can write Eq. (1) in simple terms of signal and noise statistics

$$SNR_P = \frac{\overline{s^2}}{\sigma^2_{noise}}, \quad (3)$$

where $\overline{s^2}$ is the average (expected value) of the square of the signal, and σ^2 is the variance of the noise. If we describe a signal by its statistics; mean and variance then

$$\overline{s^2} = \mu^2_{signal} + \sigma^2_{signal} . \quad (4)$$

This choice of SNR measure is based on history and the nature of the signals being measured. For audio signals, among the first to be analyzed in this way, the information is conveyed as a time-varying signal with an average value of zero, e.g., as shown in Fig. 1. This example shows the beginning of the recording. The first part is nominal silence so the observed fluctuations are noise. The piano music starts at the 1.5 secs. mark. Simply put, we hear *changes* (variation) in sound pressure, not its *average* or constant value. It was natural, therefore to have defined a signal statistic as one based on variation, such as the mean-squared value (and variance), rather than its average value. The average value would not be useful, and have a value of zero, anyway.

For this type of zero-mean signal, from Eqs. (3) and (4)

$$SNR_P = \frac{\sigma_{signal}^2}{\sigma_{noise}^2} . \qquad (5)$$

Now we turn to signals carry information based on their *value* (amplitude) rather than variation.

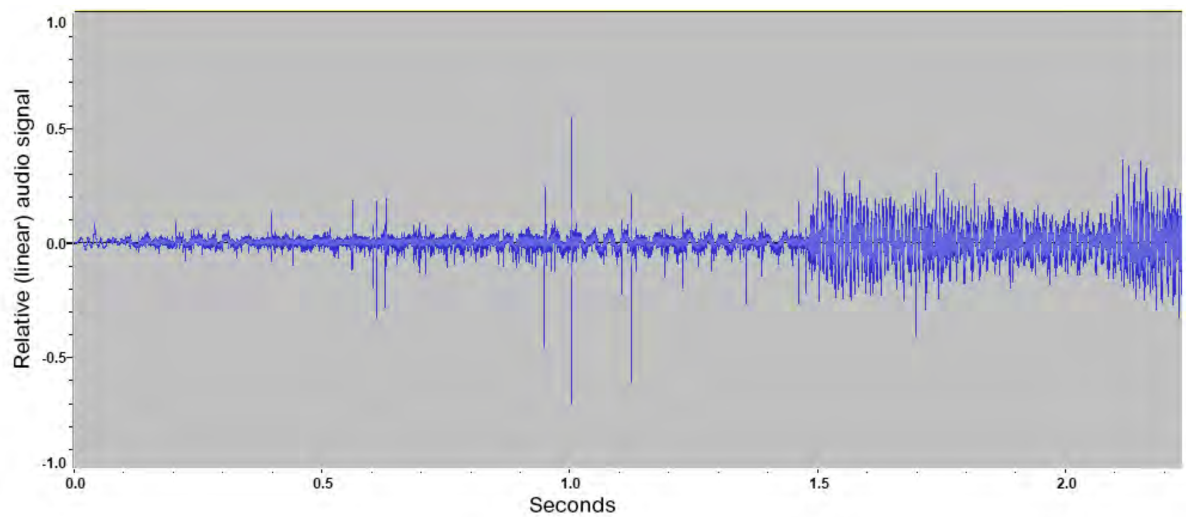

Figure 1: Example audio signal, courtesy of [Internet Archive]

What about Imaging?

For digital imaging, when we discuss SNR as captured by a detector we often compare SNR results with, and as function of, input scene exposure signal rather its pixel-to-pixel variation. For these systems we make measurements for a series of nominally constant input signal values. In these cases, Eq. (3) becomes

$$SNR_1 = \frac{\mu_{signal}^2}{\sigma_{noise}^2} , \qquad (6)$$

where our analysis is restricted to simple pixel-based signal and noise statistics. An alternative SNR based on the amplitude of the signal is

$$SNR_2 = \frac{\mu_{signal}}{\sigma_{noise}} , \qquad (7)$$

the ratio of mean signal value and standard deviation of the noise variation. With the interpretation of nominally constant input signal, SNR_2 would appear to adhere to the meaning of 'signal-to-noise ratio'. This form of SNR is also a unit-less ratio. This is the definition used by the European Machine Vision Assoc. (EMVA) in their Standard 1288,[2] ISO 15739 standard [3] and many other publications.

The two SNR measures are related as

$$SNR_2 = \sqrt{SNR_1} \quad (8)$$

In summary, there are two commonly used SNR measures for historical and practical reasons. They only share a value of unity when signal and noise measures are equal, but things do get better.

Decibels (dB)

The logarithmic scale used for sound-pressure measurement was defined in terms of SNR_1, from Eq. (1)

$$SNR_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right).$$

If we adopt the SNR_1 form for digital imaging of Eq. (6)

$$SNR_{dB} = 10\log_{10}\left(\frac{\mu_{signal}^2}{\sigma_{noise}^2}\right). \quad (9)$$

To find the equivalent measure in dB based on SNR_2, using the property of logarithms and Eq. (8)

$$SNR_{dB} = 20\log_{10}\left(\sqrt{\frac{\mu_{signal}^2}{\sigma_{noise}^2}}\right)$$

$$= 20\log_{10}\left(\frac{\mu_{signal}}{\sigma_{noise}}\right) \quad (10)$$

So the right-hand side of Eqs. (9) and (10) give the same result.

Conclusion

If you are using the SNR_1 based on squared signal and variance, multiply the logarithm by 10. If using the (mean) signal amplitude and standard deviation, SNR_2 multiply by 20. The results will be consistent. ***But,*** if you are interested in recommendations based on, e.g., acceptable SNR (ratio) values of 5:1, 40:1 etc., ask which SNR definition is being used. Visual examples can be helpful in interpreting results, and requirements for particular applications and tasks. *Forewarned is forearmed.*

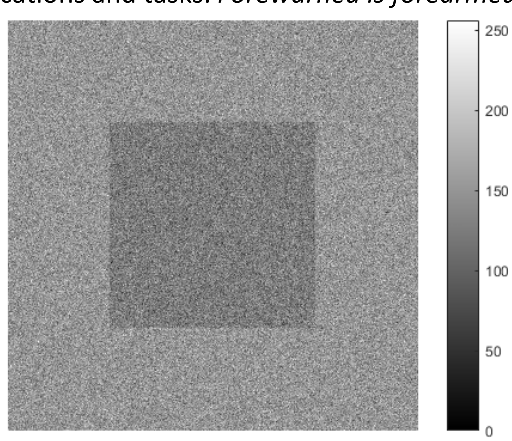

Figure 2: Example with SNR = 1 (0 dB) from Part 2 of the report

References

1. Arthur Smith and his Cracker Jacks, *Guitar and Piano Boogie,* 1950, transferred from 78-rpm record, Internet Archive
2. European Machine Vision Assoc. (EMVA) Standard 1288
3. ISO 15739:2017 Photography — Electronic still-picture imaging — Noise measurements

Coming soon:

Image Signal-to-Noise Ratio– Part 2
What does x dB of SNR look like, and when does it break down?