Error Analysis for Digital Image Acquisition and Signal Processing

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Abstract

All imaging systems are subject to error sources, and detected signals can include variation because of detector and calibration errors. For large populations, it is often assumed that the error can be modeled as a random variable having a zero mean. However, in the case of a single color instrument, camera, or scanner, error caused by deterioration of a physical standard, optical filter, or detector can introduce a bias into the measurement or image data.

During electronic image acquisition, optical scene information is not only detected but also converted into a digital form for further image processing and exchange. These signal-processing steps transform image signals as well as their image noise statistics. An important factor in the performance of image processing algorithms and color transformations is their susceptibility to error. Analysis of this can be used in optimizing integrated systems. We describe how calibration error can be combined with error-propagation methods to predict the bias-error characteristics in the stored or processed image. This approach is related to the propagation of noise variance and covariance.

Bias Error

Error propagation usually refers to the transformation of errors that occur whenever we transform a signal. When error sources can be viewed as random variables, we are often interested in error statistics, rather than complete distributions. In this case, approximations can be applied to common color-signal transformations used in digital imaging systems. Image-noise variance originating at a detector, for example, can be propagated into corresponding statistics for a transformed image.[1] Here, we address the case of a consistent bias caused by color calibration or quantization error.

If an observed signal is subject to error, it can be expressed as the sum of true value and bias,

$$\hat{x} = x + b_x ,$$

where $\hat{x}$ is the observed value, x the true value, and b_x the bias error.

For color images a transformation can be written in vector-matrix notation as

$$\mathbf{y} = \mathbf{f}(\mathbf{x}), \tag{1}$$

where

$$\mathbf{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}},\; y = [y_1, y_2, \cdots, y_m]^{\mathrm{T}} .$$

The bias in each component signal of **x** is written as a vector

$$\mathbf{b_x} = [b_{x_1}, b_{x_2}, \cdots, b_{x_n}]^{\mathrm{T}} .$$

Using a Taylor series expansion, we can approximate the propagation of bias error to the transformed color signals

$$\mathbf{b_y} \cong \mathbf{J_f} \mathbf{b_x} , \tag{2}$$

where

$$\mathbf{J_f} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \ddots & & \\ \vdots & & \ddots & \\ \frac{\partial y_m}{\partial x_1} & & & \frac{\partial y_m}{\partial x_n} \end{bmatrix}, \tag{3}$$

and each element of $\mathbf{J_f}$ is evaluated at $(m_{x_1}, m_{x_2}, \cdots, m_{x_n})$.

XYZ to CIELAB. A common color transformation is that from tristimulus values (*X,Y,Z*) to CIELAB coordinates (*L**, *a**, *b**). This can be viewed as a nonlinear transformation, followed by a weighted summing. The combination of these two steps can easily be analyzed, however, using the above method. Here, we assume that a bias error in the measured tristimulus values is that evident after the division by those of the white reference, (X_n, Y_n, Z_n). For values of X / X_n , Y / Y_n , $Z / Z_n > 0.00886$ the derivative matrix is

$$\mathbf{J} = \begin{bmatrix} 0 & \frac{116}{3m_Y^{2/3}} & 0 \\ \frac{500}{3m_X^{2/3}} & -\frac{500}{3m_Y^{2/3}} & 0 \\ 0 & \frac{200}{3m_Y^{2/3}} & -\frac{200}{3m_Z^{2/3}} \end{bmatrix}. \tag{4}$$

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McDowell[2] gave several 'rules of thumb' for the propagation of reflectance-factor, measurement error to CIELAB. Interpreting these instrument errors as bias introduced into the X, Y, Z values, the results of our error-propagation analysis can be compared with McDowell's computed results. This was done and the results do predict the actual measurement results. Figure 1 shows the ΔE^*_{ab} that results from a 2% nonselective bias,

$$b_X = 0.02 m_X, b_Y = 0.02 m_Y, b_Z = 0.02 m_Z,$$

plotted as an a^*-b^* surface for $L^* = 50$.

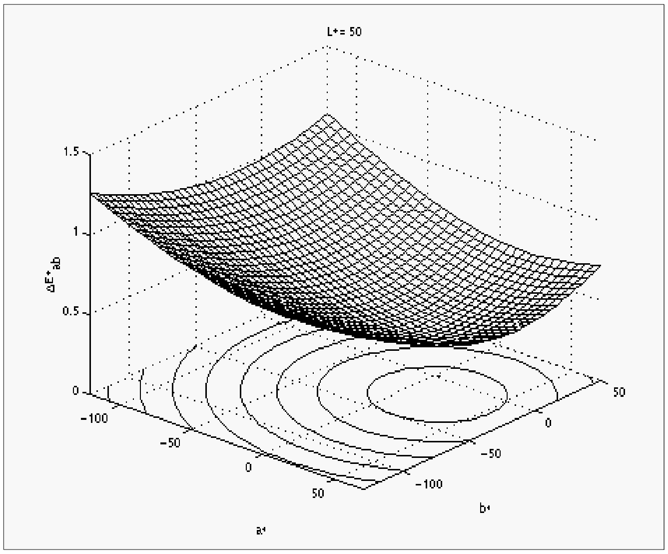

Fig. 1. ΔE^*_{ab} for 2% bias in X, Y, Z for L* = 50

CRT Gamma. A common color-encoding specification for the interchange of digital images is sRGB.[3, 4] Developed to facilitate viewing of images on computer monitors, it includes the color characteristics of a reference monitor. For an input signal, d [0-1], the resulting CRT luminance factor is[5]

$$I = (k_1 d + k_2)^g, \quad (5)$$

where k_1 and k_2 are the system gain and offset and γ is the CRT gamma. If a monitor deviates from expected performance by a bias in the gamma value, the bias propagation is

$$b_I \cong \left(d^{m_g} \log_e d\right) b_g, \quad (6)$$

where k_1 and k_2 have been set to 1 and 0. For a true value of $m_g = 2.2$ and various levels of bias, the luminance factor bias was computed using Eq. (6).

The derivative matrix of Eq. (4) was then used to propagate the luminance factor error to CIELAB. The resulting bias in L^* is plotted in Figure 2. This agreed with direct calculation of the difference error.

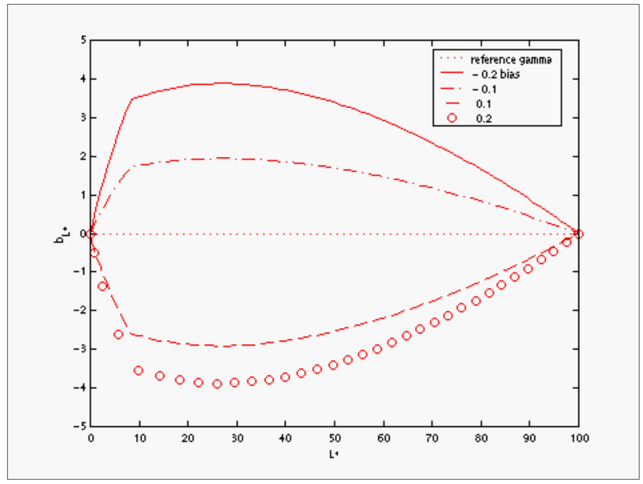

Fig. 2. Bias in L* caused by bias in γ, in L* units [0-100].

Conclusions

Statistical-error propagation provides a tool for the prediction and understanding of both image noise- and color- calibration bias error in digital imaging systems. The method can be expected to be useful in color measurement and calibration performance for a wide range of practical transformations.

References

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