Signal-To-Noise Ratio Analysis Of Charge-Coupled Device Imagers

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Abstract

We start with a physical model[1] of the detector and associated electronics. Using previously derived expressions for the MTF and noise power spectrum, we extend the use of the model to include a signal-to-noise metric, the noise equivalent quanta (NEQ). This approach is then applied in a design example relevant to film and document scanning.

1. Introduction

Charge-coupled device (CCD) sensors are now the detector of choice for many electronic imaging applications. For integrated systems, understanding the imaging characteristics of the acquisition step is important since they limit the image information available for processing and display. System design not only involves the integration of various technologies to perform different tasks, but also the comparison of and selection between competing technical options for the same task. A consistent systematic description of signal modulation and noise degradation is valuable when identifying limitations to image quality. In particular, signal-to-noise ratio (SNR) techniques aid in the comparison of competing technologies, and in the evaluation of ultimate performance.

The SNR requirements for an image acquisition subsystem are set by the intended application. Determination of these requirements involves consideration of both the input scene information and subsequent image processing and display. For the evaluation of the efficiency with which image information can be acquired the detective quantum efficiency (DQE) is often used.[2] The corresponding description of the SNR, for quantum-limited applications, is the noise equivalent quantum (NEQ) exposure.[3] Although these metrics were originally applied to photographic film and video cameras, the NEQ has also been used to describe the imaging characteristics of, for example, electrophotographic halftone printing,[4,5] laser printers,[6,7] astronomical detectors[8] and medical diagnostic systems.[9,10] The noise equivalent input approach has also been applied to the performance of human vision.[11] The NEQ describes the absolute SNR in terms of an equivalent input quantum exposure. It can, therefore, be used to compare imaging systems and components that use different technologies and physical mechanisms. Here we only address CCD imagers with the goal of expressing the NEQ in terms of the various model design parameters.

2. CCD Imager Model

Figure 1 shows a functional block diagram of a CCD sensor in the focal plane of an optical system and its associated electronics. Incident photons are detected in the photosensitive layer simultaneously at each photosite (pixel) location across the detector. The energy released by the absorbed photons generates electron-hole pairs. The electrons then migrate to potential wells in the device so that the collected charge represents the signal integrated over each photosite. This charge, collected during a fixed time interval, is then transferred from the potential wells through a shift register to the readout node. The final step is the readout of the charge packets, analog amplification and quantization of the signal. A previously described model[1] addresses the imaging characteristics of a CCD imager in terms of these physical steps. As each step of the imaging system is examined, the signal and noise characteristics are expressed in terms of the resulting MTF and Wiener, or noise power, spectrum. We briefly describe the model in order to derive expressions for the NEQ. More details can be found in Ref. 1.

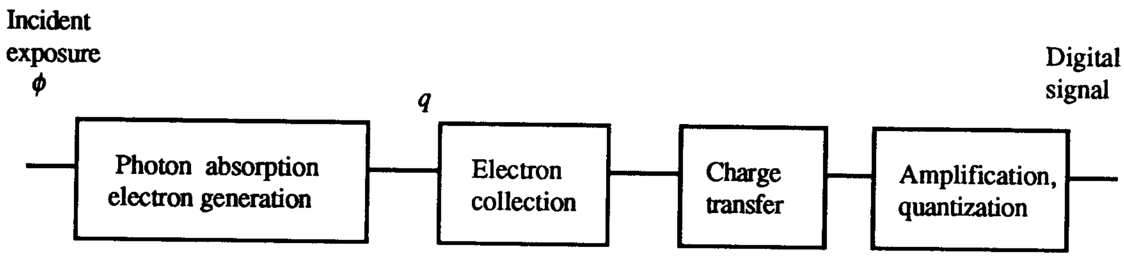

Figure 1. Functional diagram for CCD imager.

For a uniform exposure of μ_ϕ photons/area the fluctuations are assumed to be governed by Poisson statistics so the variance is

$$\sigma_\phi^2 = \mu_\phi$$

We will restrict our analysis to visible light wavelengths, where at most a single electron-hole pair is generated for each interacting photon.[12]

Not all photons are absorbed by the photosensitive material, and not all generated electrons are collected. The absolute quantum efficiency is $\eta(\lambda)$, where λ is the wavelength of the exposing radiation. The variance in the number of collected electrons is

$$\sigma_q^2 = \mu_q = \eta\, \mu_\phi \quad . \tag{1}$$

The diffusion of charge between photosites is assumed to be a scattering process prior to charge collection. This leads to a blurring of the signal but no change in the variance, or noise power spectrum of the image noise for a uniform illumination. The integration of the signal charge over each photosite can be described by the introduction of an additional MTF into the signal path, which for a rectangular photosensitive area, is

$$H_i(u,v) = \frac{sin(\pi u \Delta x)}{\pi u \Delta x}\,\frac{sin(\pi v \Delta y)}{\pi v \Delta y} \,, \tag{2}$$

where Δx and Δy are the dimensions of the active pixel area. The diffusion and integration modulation transfer functions combine[13] so the MTF associated with diffusion and integration is

$$H_{d\text{-}i}(u,v) = H_d(u,v)\;\; H_i(u,v) \quad . \tag{3}$$

The integrated pixel charge is Poisson distributed since each is the sum of independent Poisson random variables.[14] The spectrum shape remains unchanged since the pixel integration is over nonoverlapping regions containing the uncorrelated charge fluctuations so the noise power spectrum is

$$S_i(u,v) = \mu_i \qquad \text{electrons}^2 \cdot \text{pixel} \;\; . \tag{4}$$

Since some charge is lost during each transfer, this step can contribute a blurring of the signal[15,16] and the introduction of image fluctuations.[17] These effects are minor, however, for current buried-channel CCDs. The noise power spectrum for the transferred signal is

$$S(u,v) = S_i(u,v)\ H_t^2(u,v) + S_\varepsilon(u,v) \quad , \tag{5}$$

where H_t and S_ε are the MTF and noise power spectrum due to charge transfer inefficiency.

Two other noise sources are important to our analysis of the CCD imager: fixed-pattern noise and read noise. We will assume that fixed-pattern noise is corrected for by calibration and digital post processing of the signal. The read noise, which is assumed to be independent of the mean signal, results from several noise sources.[17,18]

The on-chip output amplifier contributes both an uncorrelated white noise and a source with a 1/f spectral density. We model the result, as in Ref. 12, as the addition of these two sources followed by a linear filter that includes the response of any correlated double sampling operation.[19] The MTF from input to amplifier and noise power spectrum at the output of the amplifier are

$$H_{d\text{-}a}(u,v) = H_d(u,v)\ H_i(u,v)\ H_t(u)\ H_a(u)\ , \tag{6}$$

$$S_a(u,v) = \left\{S_t(u,v) + S_r(u,v)\right\} H_a^2(u) \ \cdot \tag{7}$$

We model the process of signal quantization as the addition of a zero-mean quantization noise source. Making the usual assumption of a uniform distribution of error about a given quantization interval, the variance for the case of uniform quantization intervals is[20]

$$S_b(u,v) = \frac{q^2_{max}}{12\ 2^{2b}} \qquad \text{electrons}^2\text{pixel} \quad , \tag{8}$$

where q_{max} is the maximum pixel charge, and b is the number of bits used.

Each step in the signal chain has now been addressed. The system MTF is the product of each component MTF from equations (2), (3), (5) and (6):

$$MTF(u,v) = H_d(u,v)\ H_i(u,v)\ H_t(u)\ H_a(u)\ . \tag{9}$$

The output Wiener spectrum, from equations (4), (5), (7) and (8) is

$$S(u,v) = \left\{\mu_i\, H_t^2(u) + S_\varepsilon(u,v) + S_r(u,v)\right\} H_a^2(u) + S_b(u,v)\ . \tag{10}$$

3. Noise Equivalent Quanta

The NEQ is the output SNR expressed as an equivalent input quantum exposure, that would be required by an ideal detector, to yield the same SNR. For any practical image acquisition step, the NEQ is some fraction, [0-1], of the actual exposure. This fraction is the DQE since

$$DQE = \frac{NEQ}{\mu_\phi}\ .$$

The NEQ for the detected image is[3]

$$NEQ = \frac{\mu_\phi^2\ MTF^2\ G_d^2}{S_\phi} \tag{11}$$

where G_d is the mean level transfer gain, $d\phi/do$, and o is the output signal. For our application, o, might be expressed as digital signal value or number of electrons.

From equation (11) we need to know the mean exposure, MTF and output noise power spectrum. In some cases we do not know the absolute quantum exposure, but can take advantage of the uncorrelated quantum nature of the generated charge. Here we assume that at most one electron is generated for each absorbed photon ($\eta \leq 1$). We can treat the CCD imager as a lossy photon counter. We note that the fixed-pattern noise correction is assumed to have subtracted the mean dark offset charge. If this is the case then the relationship between average exposure and average generated charge in electrons is as in equation (1)

$$\mu_\phi = \frac{\mu_q}{\eta} \tag{12}$$

We can, therefore, express the CCD imager gain as

$$G = \frac{d\mu_b}{d\mu_a} \frac{d\mu_a}{d\mu_t} \frac{d\mu_t}{d\mu_q} \frac{d\mu_q}{d\mu_\phi} \tag{13}$$

where the subscripts, q, t, a, and b, indicate the signal in terms of generated electrons, transferred electrons, amplified voltage (or current), and quantized digital value, respectively. The image charge loss mechanisms that occur after electron generation have all been assumed to affect image detail (described by MTFs and noise sources), but not the mean level, therefore $dt/dq = 1$. Equations (12) and (13) can now be substituted into equation (11) to express the NEQ at the output of the CCD imager. From equations (9), (10) and (11)

$$NEQ = \frac{\{\mu_\phi H_d\, H_i\, H_t\, H_a\, G\}^2}{\{\mu_i\, H_t^2 + S_t + S_r\}\, H_a^2 + S_b} . \tag{14}$$

It should be emphasized that the output noise power spectrum of the denominator must be expressed in terms of output signal units consistent with the gain, G. For example, if the noise power spectrum is expressed as digital values from the A/D converter (as is often done for RMS noise measurement), the gain must be expressed as in equation (13), $G = d\mu_b / d\mu_\varphi$. A more common practice when device physics is being addressed, is to express the output signal and noise measurements in terms of the effective number of electrons. These characteristics can easily be used to determine the output NEQ. In this case the gain is merely given by

$$G = \frac{d_t}{d_\phi} = \eta \tag{15}$$

since dt/dq is assumed to be unity. From equations (14) and (15)

$$NEQ = \frac{\{\mu_q\, MTF\, \eta\}^2}{\eta^2 \quad S}$$

or

$$NEQ(\mu_q,u,v) = \frac{\{\mu_q H_d\,(u,v)\;\; H_i\,(u,v)\, H_t\,(u)\;\; H_a\,(u)\}^2}{\{\mu_i\, H_t^2(u) + S_e(u,\,v) + S_r\,(u,\,v)\}\, H_a^2(u) + S_b(u,\,v)} \tag{16}$$

where the output noise power spectrum is expressed in units of electrons^2 area (i.e., variance/spatial frequency^2).

4. Computed Example

We will now apply the above expression for the NEQ in a design example. We wish to evaluate the SNR performance of a linear CCD imager to be used to scan photographic film or printed images. The manufacturer specifies that the maximum charge is 10^5 electrons and the dynamic range (q_{max}/RMS pixel dark noise) is 1500. The image signal is to be quantized using 8 bits and the effective sampling distance is 100 μm. We can assume that the charge transfer efficiency is high (H_t= 1 and S_ε = 0), and the amplifier 1/f noise is negligible when compared to other read noise sources.

The imager MTF is the product of those for the charge integration, diffusion, and amplifier steps. The integration MTF is given by equation (2), and the one-dimensional diffusion MTF is assumed to be of Gaussian shape. The amplifier MTF will be assumed to be described by a simple low-pass filter

$$H_a(u) = \frac{1}{1+\frac{u}{7}} .$$

where u is in cy/mm on the input document. The combined system MTF is shown in Fig. 2.

The read noise power spectrum, from equation (12) is

$$S_r(u,v) = \left(\frac{q_{max}}{DR}\right)^2 H_a^2(u)$$

$$= 4.44 \times 10^3 H_a^2(u) \quad \text{electrons}^2 \text{ pixel,}$$

where DR is the dynamic range.The shot noise power spectrum for a mean charge, μ_q,is $\mu_q H_a^2(u)$ electrons^2 pixel. For uniform quantization, from equation (8) is

$$S_b = \frac{q_{max}^2}{12\ 2^{16}} = 1.27 \times 10^4 \quad \text{electrons}^2 \text{ pixel}$$

The noise power spectrum is the result of the shot, read, and quantization noise components, as in equation (10).

The output NEQ can be found from equation (16), and is shown in Fig. 3 plotted versus spatial frequency and mean relative exposure. To plot the function the single frequency axis corresponds to $u = v$. The dashed lines of Fig. 3 indicate the limiting NEQ due to the number of collected electrons, i.e., the shot noise limit. For low exposure signals the quantization error represents a significant noise component. If the A/D converter is preceded by a nonlinear analogue amplifier, nonuniform quantization can be achieved. If for the previous case we assign the 8 bit levels equally spaced in the logarithm of the signal over the range signal range [0 01 - 1.0], then the resultant NEQ surface is shown in Fig. 4. A comparison of Figures 3 and 4 reveals that the log-spaced quantization resulted in higher NEQ for low exposures at the expense of the maximum NEQ. This is consistent with putting the most quantization error in regions of high signal fluctuations. The benefit of reduced read noise in terms of NEQ can also be calculated. If we now assume that our CCD imager has a dynamic range, with the output 8 bits log-quantized then the result is shown in Fig. 5. In this case the NEQ was not increased substantially over that in Fig. 4, except at low exposures, as we might expect. The entire NEQ surface could be increased by increasing the maximum signal charge, q_{max}, or scanning with smaller sampling interval, which also increases the information bandwidth.

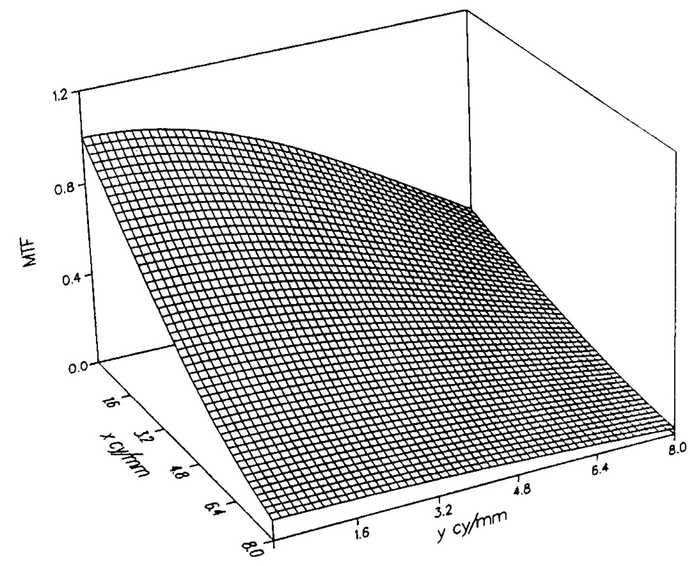

Figure 2. CCD imager MTF for the computed example.

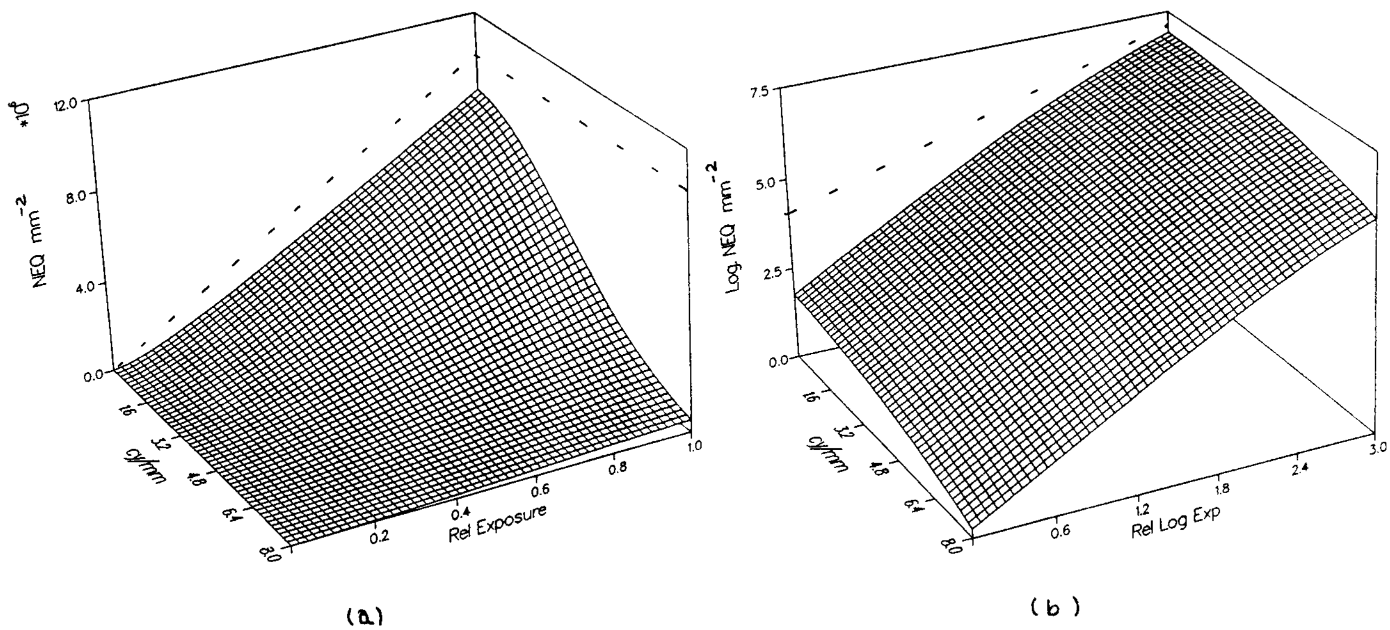

Figure 3.Output NEQ surface for the computed example and uniform quantization plotted as (a) NEQ versus spatial frequency and relative exposure, and (b) Log NEQ versus spatial frequency and relative log exposure.

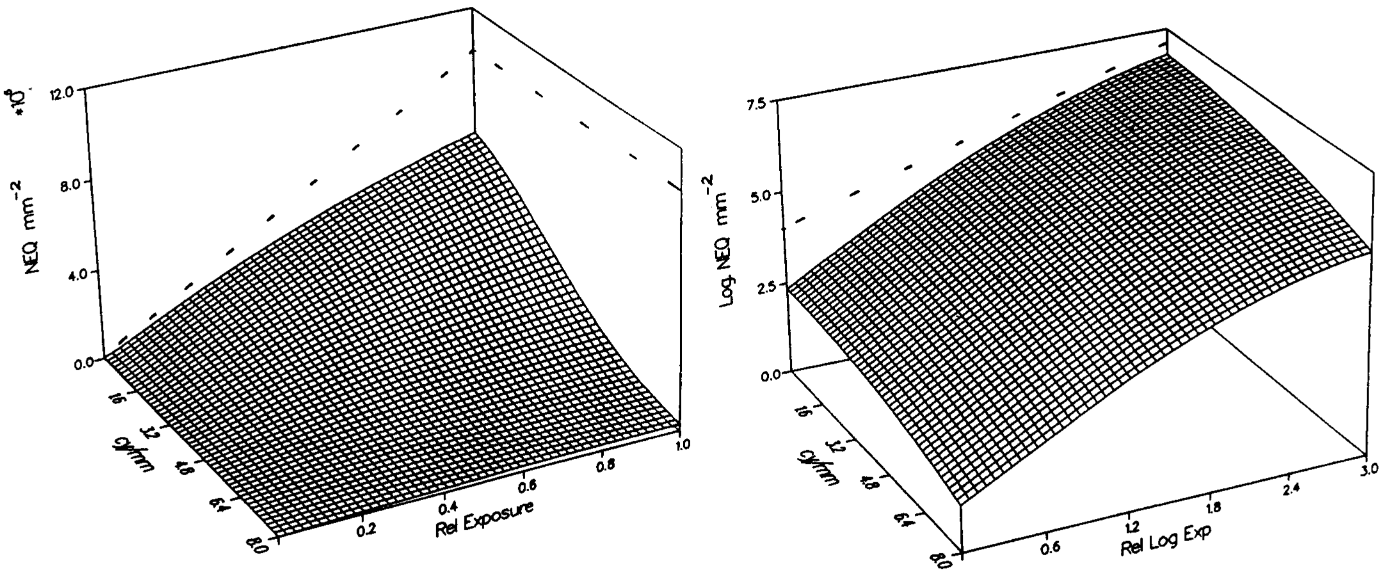

Figure 4. Output NEQ surface for the example with log-spaced quantization intervals over the relative exposure range [0.001-1.0].

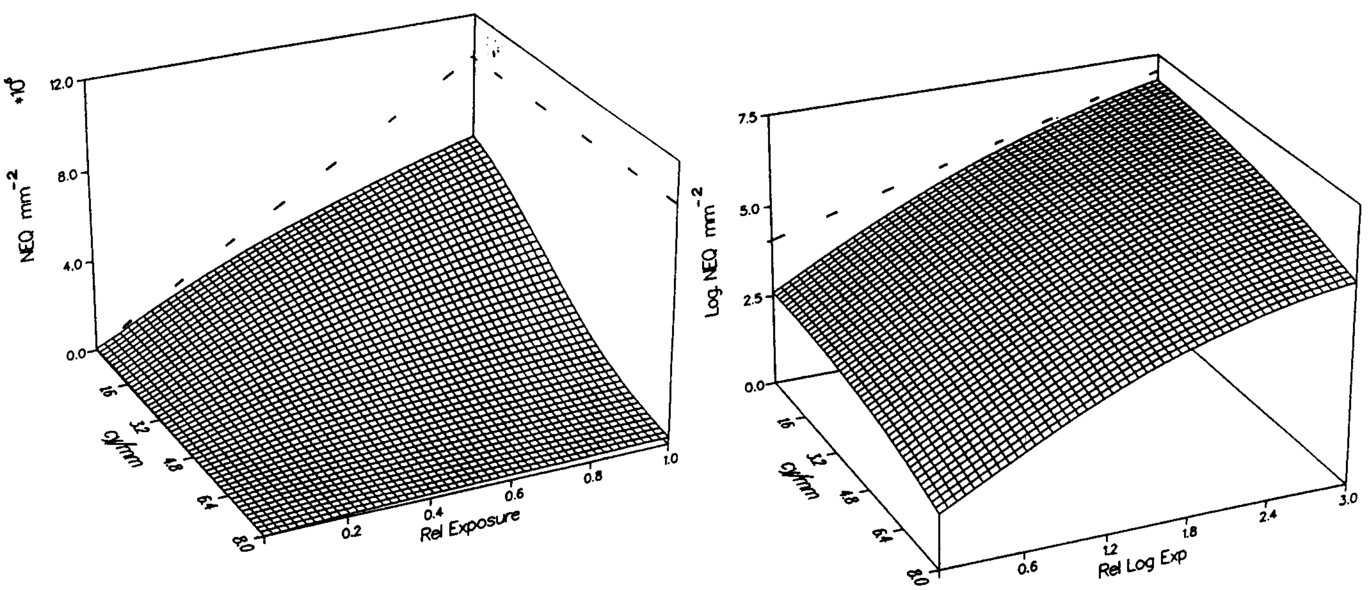

Figure 5. Output NEQ surface for the example, but with an increased dynamic range of 2000 (lower read noise) and log-spaced quantization intervals.

5. Conclusions

The SNR comparison of various imaging technologies is facilitated by the use of the NEQ which, for a quantum-limited exposure is equal to the square of the SNR. Here we have used the model for a CCD imager to derive an expression for the output NEQ in terms of several design parameters. The functional form of the model facilitates the evaluation of subsystem performance in terms of established measures, such as DQE and NEQ, and image simulation, which could also include scanning artifacts.

The computed example indicates how this tool might be used. In this way we can quantify potential improvements to the available image SNR from alternative designs. In addition, since actual image acquisition subsystems introduce additional degradation, we can evaluate their SNR against limiting performance.

6. References

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