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Particulate model for halftone noise in electrophotography
I. Theory

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Abstract

A model is constructed for the noise-density characteristics of electrophotographic halftone images. The model has components relating to the fluctuations: within the dot; within the background area; and in dot-size. These components are themselves related to the (toner) particulate nature of the image, enabling comparison to be made with the equivalent continuous tone case.

Introduction

During recent years an increasing amount of attention has been paid to the role of image noise in electrophotography[1-6] Investigations have covered both physical and psychophysical aspects, and have included measurement techniques, sub-system analysis, and particulate models. Most of these investigations have concerned themselves with the fluctuations on nominally uniform image areas. Less is known about the nature of the fluctuations arising in electrophotographic halftones, and the purpose here is to construct a particulate model which relates in detail the noise in halftones to that associated with continuous tones. A model has previously been proposed[1] which works well but which is based only on intuitive reasoning.

By use of the term noise for halftones we imply the image fluctuation in nominally uniform areas. However, these fluctuations are observed in the electrophotographic reproduction of noise-free halftones in the original. We are thus not concerned with the periodic fluctuations arising from the patterning of the halftone itself, but with the random fluctuations due to the natural statistical processes associated with electrophotography. The period of the halftone is assumed to be at a frequency beyond that of prime interest, for example typically beyond spatial frequencies which contribute to the visual impression of graininess[1] Likewise we are not here primarily concerned with either random or deterministic factors which may exist, whose influence is to modify the periodicity (dot positioning 'errors', halftone frequency variations, etc.). These will typically modify the resonant peak in the Fourier transform which is associated with the fundamental frequency by broadening or by introducing harmonics. Here we are essentially concerned with the very low frequency value of the equivalent Wiener Spectrum of the noise due to the statistical attributes of electrophotography. In general, factors which modify the periodicity may have a marked effect on the images of lines in text, or of edges in pictorial material, but will have a considerably lesser effect within nominally uniform areas.

Due to our interest in the statistical fluctuations in density from dot to dot, we are concerned with the product $A\sigma^2 = WS(0)$, i.e., the product of halftone cell area A with the mean-square density fluctuation as measured from dot to dot. This will give us an equivalent value of the low frequency Wiener Spectrum of the noise, denoted by WS(0). Further, if the statistics of dots are completely independent of those of neighboring dots, a density-measuring aperture of NA will yield the same value, i.e.,

$$NA\sigma_{NA}^2 = A\sigma_A^2$$

for any value of N, since

$$\sigma_{NA}^2 = \frac{1}{N}\sigma_A^2$$

The placement of the scanning aperture NA must however be exactly over NA complete halftone cells, and density samples thus taken by moving integral numbers of cells, typically with a scanning aperture covering 1 cell and moving 1 cell between density readings.

A Model for Fluctuations Within Dot and Background

First we consider how the joint noise obtained by dual-sampling of two adjacent noise samples is related to the two individual noise levels obtained by sampling each separately. This corresponds to the halftone problem since we are typically concerned with measuring a halftone cell which contains samples of maximum (dot) and minimum (badkground) density. Initially we ignore fluctuations in the respective sample sizes, and consider them to exist in fixed proportions.

Suppose the individual mean values of density, reflectance, and (low-frequency) Wiener Spectrum are $D_1 R_1, WS_1(0)$, and $D_2, R_2, WS_2(0)$ respectively. Let the (fixed) respective areas of the two samples be A_1 and A_2 as they appear in the joint sample A, where $A = A_1 + A_2$. Thus the fractional areas will be

$$f_1 = \frac{A_1}{A} \;, \; f_2 = \frac{A_2}{A} \;, \text{ with } f_1 + f_2 = 1 \;.$$

The problem is to form the joint values of density, reflectance and Wiener Spectrum, which we denote simply as D,R and WS(0). The reflectance will be given by appropriate linear weighting of the separate contributions:

$$R = \frac{A_1 R_1 + A_2 R_2}{A} = f_1 R_1 + f_2 R_2 \qquad (1)$$

and the density follows simply as

$$D = -\log_{10} \{f_1 R_1 + f_2 R_2\} \qquad (2)$$

The calculation of the joint Wiener Spectrum is not as straightforward. We recall that in effect we are considering the cell area A as an aperture which scans joint samples, but during scanning A_1 and A_2 have always the same fixed values. The only fluctuations thus arise from within the two individual samples (dot and background), and these individual contributions are assumed uncorrelated in their influence on the joint fluctuation. From equation (1) it follows that changes in reflectance will be related to each other according to

$$\Delta R = f_1 \Delta R_1 + f_2 \Delta R_2$$

and hence the variances will be related according to

$$\overline{\Delta R^2} = f_1^2 \overline{\Delta R_1^2} + f_2^2 \overline{\Delta R_2^2}$$

which we write as

$$\sigma_R^2 = f_1^2 \sigma_{R_1}^2 + f_2^2 \sigma_{R_2}^2 \qquad (3)$$

Since $D_1 = -\log_{10} R_1$, $D_2 = -\log_{10} R_2$, it follows that for small fluctuations

$$\sigma_{R_1}^2 = \frac{R_1^2 \sigma_{D_1}^2}{(\log_{10} e)^2} \quad , \quad \sigma_{R_2}^2 = \frac{R_2^2 \sigma_{D_2}^2}{(\log_{10} e)^2} \qquad (4)$$

We now relate these variances in density to the Wiener Spectrum value, by assuming that

$$A_1 \sigma_{D_1}^2 = WS_1(0) \quad , \quad A_2 \sigma_{D_2}^2 = WS_2(0) \quad ,$$

$$(A_1 + A_2)\sigma_D^2 = A\sigma_D^2 = WS(0) \qquad (5)$$

Combining equations (3), (4) and (5) we arrive at

$$WS(0) = \frac{WS_1(0)}{f_1 \left(1 + \frac{f_2 R_2}{f_1 R_1}\right)^2} + \frac{WS_2(0)}{f_2 \left(1 + \frac{f_1 R_1}{f_2 R_2}\right)^2} \qquad (6)$$

The assumption implicit in equation (5) is that the effective apertures A_1 and A_2 are large enough to act as filters of all but low spatial frequencies. This will depend on the shapes of the respective Wiener Spectra $WS_1(w)$ and $WS_2(w)$, and will hold exactly for flat spectra. It will hold less exactly the more the spectra are rapidly decreasing functions of frequency. In the latter cases the product of aperture and mean-square fluctuation will be greater than the low frequency value of the Wiener Spectrum, and may be thought of as an upper limit. Similarly equation (6) will then represent an upper limit for WS(0), and in general the joint noise will be less than that predicted from the Wiener Spectra of the constituents.

We can use equation (6) to compare halftone noise with that associated with continuous tone noise at equivalent image density levels. For example, if we assume $WS_2(0)$, $R_2=1$ (noise-free white background) equation (6) reduces to

$$WS(0) = \frac{WS_1(0)}{f_1\{1+\frac{(1-f_1)}{f_1R_1}\}^2} \tag{7}$$

If we now invoke the Siedentopf relationship[7], assuming the halftone dots are formed of random assemblies of mono-sized particles of area a,

$$WS_1(0) = a\ \log_{10}e\ \ D_1 = a\ \log_{10}e\ \ \log_{10}(\frac{1}{R_1}) \tag{8}$$

Hence

$$WS(0) = \frac{a\ \log_{10}e\ \log_{10}(\frac{1}{R_1})}{f_1\{1+\frac{(1-f_1)}{f_1R_1}\}^2} \tag{9}$$

Equation (9) expresses the halftone image noise in terms only of the toner particle size, and the fractional area and reflectance of the dot. The equivalent expression for halftone image density level follows from equation (2) as

$$D = -\log_{10}\{f_1R_1+(1-f_1)\} \tag{10}$$

Equations (9) and (10) can thus be used to predict the noise-density characteristics, which we wish to compare with those for the equivalent continuous tone density. The latter density will be defined simply by $\log_{10}(\frac{1}{R_1})$, with the corresponding noise already defined according to equation (8). Thus for a given particle area a, we can compare the continuous and halftone noise characteristics for the assumed dot and background properties of this example.

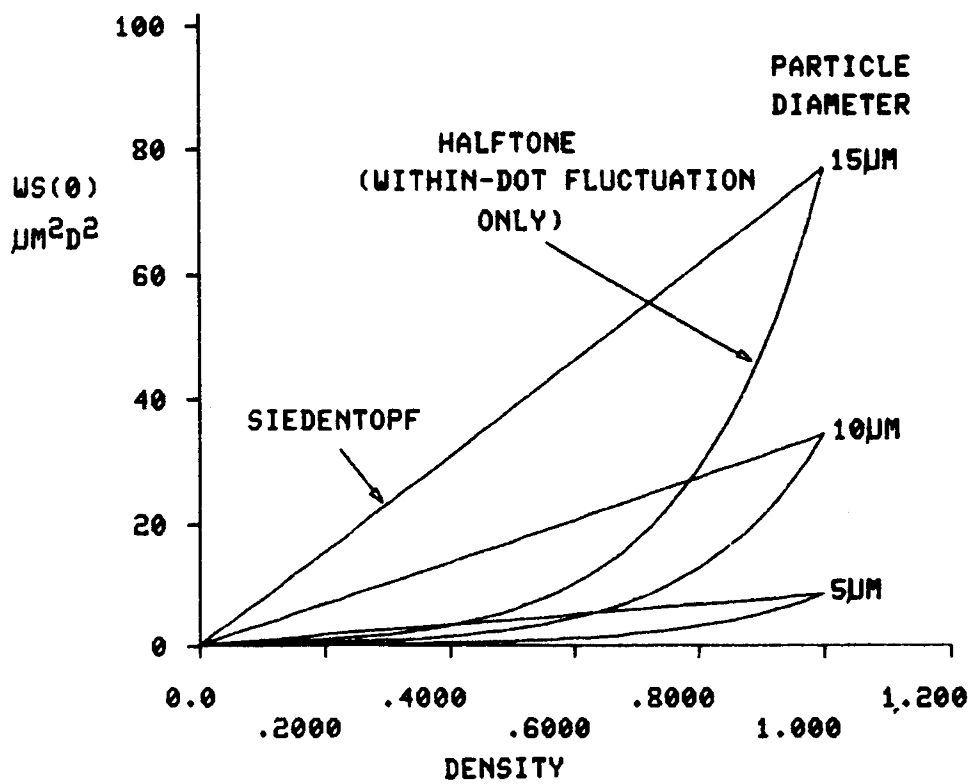

Figure 1. Comparative noise-density characteristics according to model.

Figure 1 shows computed model noise-density curves for three specific sizes of toner particle, with diameters 5, 10 and 15 μm respectively. As usual, in the random (continuous tone) case the straight-line nature of the Siedentopf equation results. However, the half-tone noise is non-linear and in general significantly less than the continuous tone noise at equivalent image density levels. The two of course coincide at maximum density, where the dot structure has disappeared ($f_1=1$). The conclusion from the relative shapes of the two sets of curves is somewhat surprising. In principle it appears that a given low image density level can be achieved using the same electrophotographic process in halftone or continuous tone modes, but with the former having associated noise levels which are orders of magnitude less than those in the latter case. This may appear to provide a potential straightforward noise-cheating technique. However we must consider the basic nature of signal reproduction by halftones. Modulation of signal is obtained via modulation of dot area, and so far we have not considered statistical noise aspects associated with the latter.

A Model for Fluctuations of Dot Size

We now allow for small variations in A_1 and A_2, subject to the same fixed cell area A. This will lead to an additional noise term, essentially due to the size variation of the dots. To do this we assume R_1 and R_2 are now fixed, and hence from (1)

$$R = R_2 - f_1(R_2 - R_1)$$

and it follows that

$$\overline{\Delta R^2} = (R_2 - R_1)^2 \, \overline{\Delta f_1^2}$$

which we write as

$$\sigma_R^2 = (R_2 - R_1)^2 \, \sigma_{f_1}^2$$

Since

$$\sigma_{f_1}^2 = \frac{\sigma_{A_1}^2}{A^2}$$ it follows that for small fluctuations

$$WS(0) = \frac{(R_2 - R_1)^2 \, (\log_{10} e)^2}{A(f_1 R_1 + f_2 R_2)^2} \, \sigma_{A_1}^2 \qquad (11)$$

The overall image noise has now been defined according to the model, and is obtained by simple summation of the two components (i.e., equations 6 and 11).

We now ask if $\sigma_{A_1}^2$ can be cast in terms of a simple particulate model, as for $WS_1(0)$ and $WS_2(0)$ in terms of Siedentopf, to enable us to make comparisons of the respective contributions to overall noise for a given toner particle size. One such model readily follows from simple geometrical considerations. We assume that

$$A_1 = n\,a \qquad (12)$$

where n is effective number of individual particle areas which make up a single dot. Thus

$$\sigma_{A_1}^2 = a^2 \sigma_n^2 \qquad (13)$$

and assuming n is a random variable from dot to dot, governed by Poisson statistics, such that $\sigma_n^2 = n$, then

$$\sigma_{A_1}^2 = a^2 n = A_1 a \qquad (14)$$

It should be noted that n will generally be much less than n_1, depending on the nature of the relationship between D_1 and n_1. If a relationship of the nature of the Nutting equation[8] holds, i.e.,

$$D_1 = \log_{10} e \, \frac{n_1 a}{A_1} \qquad (15)$$

and if D_1 is (say) 1.3, then it follows that the ratio $\frac{n_1 a}{A_1}$ will be approximately 3, whereas by equation (12) the ratio will be 1. This is no more than restating the well-known result that to obtain high density levels with a random spatial array of opaque particles requires a total particle area several times greater than the area being covered by the particles. However the assumption implicit in equations (12), (13) and (14) is that it is the statistics of an effective lower number of particles that govern the dot area fluctuations.

Substitution of equation (14) in (11) leads to

$$WS(0) = \frac{(\frac{R_2}{R_1} -1)^2 \ (\log_{10}e)^2 a}{f_1 \ (1+\frac{f_2 R_2}{f_1 R_1})^2} \tag{16}$$

Equation (16) thus achieves our aim of arriving at an expression for the dot area fluctuation in terms of the underlying particulate dimensions. There are however limitations to the application of this equation, as we shall see from the following calculations.

Numerical Example

We can now compare the fluctuations due to dot area with those previously calculated due to noise within the dot. If again for the sake of comparison we assume that $WS_2(0)=0$, $R_2=1$, equation (16) reduces to

$$WS(0) = \frac{(\frac{1}{R_1} -1)^2 (\log_{10}e)^2 a}{f_1 \{1+\frac{(1-f_1)}{f_1 R_1}\}^2} \tag{17}$$

Comparison of equations (9) and (17) shows that for a fixed particle area the noise due to dot area fluctuations will be a factor

$$\frac{(\frac{1}{R_1} -1)^2 \log_{10}e}{\log_{10}(\frac{1}{R_1})}$$

larger than that due to fluctuations within the dot. For example, for a dot reflectance $R_1=0.1$ this factor would be around 35.

Figure 2 shows the noise-density characteristics corresponding to dot-area and within-dot fluctuations, compared to the continuous tone equivalent (Siedentopf). We note that the curve for dot-area approximates to Siedentopf at low image density levels but then increases in a non-linear manner.

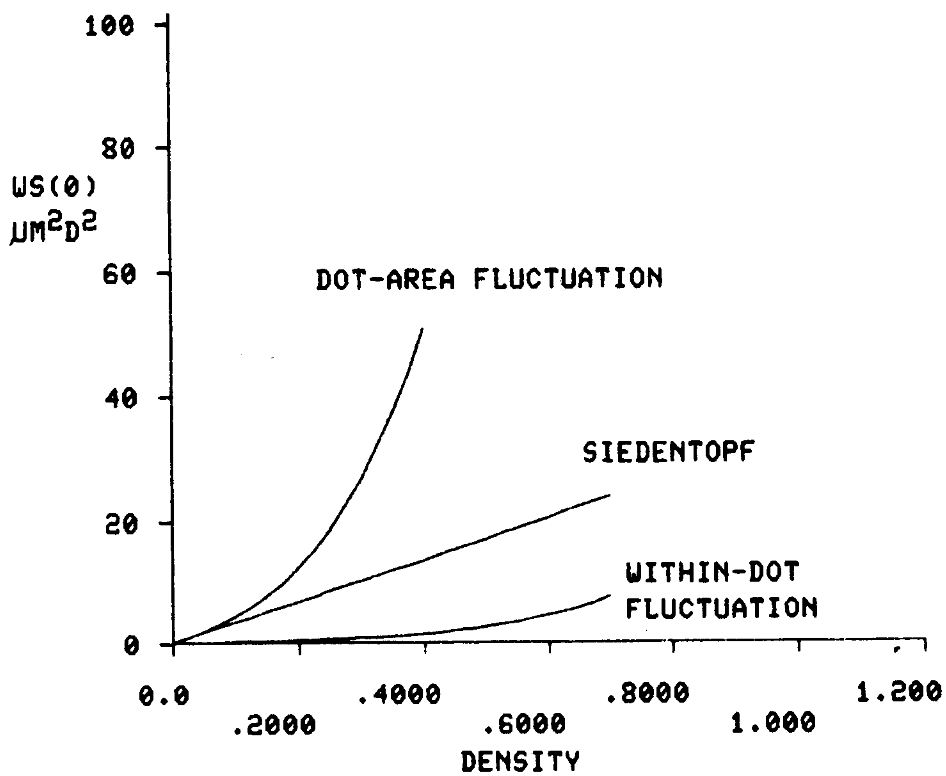

Figure 2. Comparative noise-density characteristics based on 10 μm particle diameter.

We have plotted this curve only to a density of 0.4, due to implicit practical limitations in our assumptions leading to equation (14). In fact the assumption that the area fluctuation will be driven by an unbound random variation ($\sigma_n^2=n$) in the effective number of particle areas forming the dot area, must by definition break down for larger dot sizes. In the limit, as the dot size approaches the cell size, the dot area fluctuation must approach zero. It is thus reasonable to hypothesize that the dot area fluctuation will in fact be of the form $\sigma_n^2=n\ \{1-g(f_1)\}$, where g represents some function of f_1 which tends to unity. For the calculated example a 50% dot (f_1=0.5) corresponds to a image density level of 0.26. Thus already beyond this density we might expect that the fluctuation in area might already be geometrically constrained. In any case, in practice the geometrical halftone technique will for example switch in some manner from black dots on a white background to white dots on a black background. This complication is not addressed here from the model viewpoint, but we will meet this problem again from the viewpoint of limitations on practical measurements, in the second part of this paper.

Conclusion

We have constructed a model which meets our original intent to relate in detail the noise in halftones to that associated with continuous tones. The model separates the contributions to halftone noise from within-dot and dot-area fluctuations, and allows both of these to be cast in terms of the underlying particulate structure of the dots. The domination of the contribution due to the dot-area fluctuation has been established.

The second part of this report will be concerned with verification of the model via practical measurements on electrophotographic halftones.

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